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OPTIMAL DECISIONS ON PRODUCTION AND EXPORT OF INTERRELATED PRODUCTS

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1. Introduction

Two products A and B are "interrelated" if A, the "primary" product, is a major input for the production of B, the "processed" product. Some common examples of interrelated products are: fruits and fruit juices, tobacco and cigarettes, lumber and wood products, grapes and wine, and olives and olive oil. Even though such products are regularly produced and exported by most countries, most studies of export behavior ignore the interraction and feedback effects that exist among markets for interrelated products. Instead, production and export decisions for thw two products are made independently of each other, thus resulting in policies with sub-optimal total industry profits. This was first pointed out by Bautista (1978), whose model reflected the interrelationships between products, and was thus able to estimate the magnitude of the upward bias in export supply elasticities.

This paper extends the results obtained in (Bautista, 1978) by determining optimal production and export policies for an industry of two or more interrelated products. It is assumed that some form of centralized decision making is valid and effective.[1] Policies that maximize the total industry profit are determined and evaluated, and previously developed theoretical results and empirical observations of (Bautista, 1978) are confirmed.

A number of interesting results are proved:

(i) the optimal production and export quantities are achieved when the marginal revenues between different markets are equalized.

(ii) an increase in any product's world price should trigger an increase in the exports of that product, and a simultaneous decrease in its home consumption and the exports of the other product.

(iii) a currency devaluation (i.e., increase in the exchange rate) should trigger an increase in the production and exports of the processed product, while reducing the home consumption of both products.

The paper is organized as follows: in section 2 we formulate the model for two interrelated products, under the small country assumption; the maximization conditions are derived, and then solved explicitly for the case of log-linear demand functions. In section 3 we analyze the comparative static properties of the model, and determine the effects of variations in the exogenous variables. Finally, in section 4 we discuss modifications of the model when the small country assumption is relaxed.

2. The Model

Formulation

We first define our notation. Let

[1]This could be achieved through government intervention, e.g., by withholding export licenses or quality control certificates, so as to divert the product into the domestic market.

H_x = home consumption of product x, (x = A, B).

X_x = exports of product x.

P_x = home price of product x in domestic currency.

P_x^* = world price of product x in foreign currency.

R = exchange rate.

Q_T = total available supply of the primary product A.

Q_B = total amount of the processed product B produced,

= $H_B + X_B$.

$K_B(Q_B)$ = processing cost for producing Q_B units ob B.

u = number of units of A required to produce one unit of B.

MR_x = domestic marginal revenue of product x = $P_x+H_x(\partial P_x/\partial H_x)$

MR_x^* = foreign marginal revenue of product x = $R\cdot P_x^*$

MC_B = marginal cost of B = $\partial K_B(Q_B)/\partial Q_R + u\cdot MR_A$

We assume that

$$H_A = f(P_A,P_{CA},E) \ , \ HB = g(P_B,P_{CB},E) \qquad (1a)$$

where P_{cx} is the price of consumer products competing with x, and E is the expenditure variable, with

$$f_1(=\partial f/\partial P_A) < 0 \ , \ f_2(=\partial f/\partial P_{CA}) > 0 \ , \ f_3(=\partial f/\partial E) > 0 \qquad (1b)$$

$$g_1 < 0 \ , \ g_2 > 0 \ , \ g_3 > 0.$$

We also assume that foreign demand for both products A and B is infinitely elastic (small country assumption). This assumption is relaxed in section 4.

Our problem can now be defined as follows: Given the exogenous variables $\underline{V}=(P_A^*,P_B^*,R,Q_T)$, determine the values of the decision variables $\underline{D}=(H_A,X_A,H_B,X_B,Q_B)$ in order to maximize the total industry profits $\Pi(\underline{D})$,

$$\Pi(\underline{D}) = P_A(H_B)\cdot H_A+P_B(H_B)\cdot H_B+RP_A^*X_A+RP_B^*X_B-K_B(Q_B) \qquad (2)$$

subject to the constraint

$$H_A+X_A+\upsilon Q_B = Q_T \qquad (3)$$

The revenue side of the profit function is straightforward: total revenues come from four different markets, namely the domestic and foreign markets for products A and B. The cost side represents the processing cost of B, $K_B(Q_B)$, which includes labor, processing, packaging, marketing costs and depreciation. We note that the cost of producting the quantity Q_T of the primary product in the first place is omitted, since it is assumed to be exogenously given. If this assumption is relaxed, however, the model can be easily extended to hold.

It is further assumed, as in (Bautista, 1978), that B's production technology is of the fixed proportion type: u units of A are needed to produce 1 unit of B.

Profit Maximizing Conditions

By forming the Lagrangian and applying the Kuhn-Tucker conditions, it can be shown (Prastacos and Xafa, 1978) that the first order conditions for maximization reduce to:

$$MR_A = MR_A^* \qquad (4)$$

$$MR_B = MR_B^* = MC_B \qquad (5)$$

Equation (4) and the first equation of (5) state the familiar result that, given a quantity of a product to be allocated between the domestic and foreign markets, the optimal allocation is such that the marginal revenues from the two markets are equalized. The second equation of (5) relates to the special nature of interrelated products; it states that the amount of the primary product A to be used as an input for the processed commodity B should be such that the marginal revenue of B equals the marginal cost of B. This marginal cost includes the marginal processing cost plus the marginal revenue foregone by using u units of A (each of which could sell for RP_A^*) in order to produce one unit of B.

The three profit maximizing conditions together with constraint (3) can be solved for H_A, H_B, X_A, X_B, Q_B to determine the optimum market policy.

Solution for Log-Linear Demand

We now solve the model for the special case in which the domestic demand functions for A and B are log-linear, and the processing cost function for B is quadratic (Bautista, 1978):

$$\log H_A = \log \alpha_o + \alpha_1 \log P_A \; ; \; \alpha_1 < 0$$

$$\log H_B = \log \beta_o + \beta_1 \log P_B \; ; \; \beta_1 < 0$$

$$K_B(Q_B) = c_o + c_1 Q_B + c_2 Q_B^2 \quad ; \; c_1 . c_2 > 0$$

For reasons of mathematical convenience, we define the decision variables here to be P_A, P_B and Q_B. From these, H_A, X_A, H_B and X_B can then be determined.

Solving (4), (5), and realizing that $\partial P_A/\partial H_A = \alpha_1^{-1} P_A/H_A$, it follows that for optimality,

$$P_A = RP_A^*/(1+\alpha_1^{-1}) \; , \; P_B = RP_B^*/(1+\beta_1^{-1}) \qquad (6)$$

$$Q_B = (R(P_B^* - uP_A^*) - c_1)/2c_2 \qquad (7)$$

The remaining decision variables can then be determined. We note that since in the case of log-linear demand α_1 and β_1 are the elasticities of home demand for A and B, (9) and (10) represent the familiar result that

$$MR_x = P_x(1+e_x^{-1})$$

3. Comparative Statics

We now examine the comparative statis properties of the model. Table 1 shows the changes in the optimal value of each of the decision variables

D_i associated with a marginal change in each of the exogenous variables V_j. The derivations are given in (Prastacos and Xafa, 1978).

Table 1 - Comparative Statics for the log linear model: Small Country Case

	$\partial D_i/\partial V_j$	Exogenous Variables, V: P_A^*	P_B^*	R
Decision Variables, D	P_A	$R/(1+\alpha_1^{-1})>0$	0	$P_A^*/(1+\alpha_1^{-1})>0$
	P_B	0	$R/(1+\beta_1^{-1})>0$	$P_B^*/(1+\beta_1^{-1})>0$
	Q_B	$-uR/2C_2<0$	$R/2C_2>0$	$(P_B^*-uP_A^*)/2C_2>0$
	H_A	$\alpha_1H_A/P_A^*<0$	0	$\alpha_1H_A/R<0$
	H_B	0	$\beta_1H_B/P_B^*<0$	$\beta_1H_B/R<0$
	X_A	$\alpha_1H_A/P_A^* +$ $+u^2R/2C_2>0$	$-uR/2C_2<0$	$-\alpha_1H_A/R -$ $-u(P_B^*-uP_A^*)/2C_2$
	X_B	$-uR/2C_2<0$	$R/2C_2 -$ $-\beta_1H_B/P_B^*>0$	$(P_B^*-uP_A^*)/2C_2 -$ $-\beta_1H_B/R>0$

Examining first the effects of a change in the world price P_A^* of A, it is clear that an increase in P_A^* will tend to raise the exports X_A of A. This increase is absorbed by a reduction in the home consumption H_A of A as a final product, and a simultaneous reduction in the availability of A as an input for B, which implies a reduction in the quantity Q_B of the processed commodity produced.

It is interesting to note that an increase in P_A^* will have no effect on the home consumption H_B of B, and that the reduction in Q_B is fully reflected in an equal reduction in the exports X_B of B. Even though this may seem counterintuitive, its explanation is simple: we have assumed an infinitely elastic foreign demand for B. Therefore the foreign marginal revenue of B, $MR_B^*=R\cdot P_B^*$, is constant and independent of X_B. We have also assumed that the domestic marginal revenue of B, MR_B, is continuously decreasing with H_B. It is clear now that, under a reduction of Q_B, it is more profitable to reduce X_B instead of H_B, since every one of the H_B-units offers at least as big a marginal revenue as every one of the X_B-units. Mathematically, the only way to retain the optimality condition $MR_B=MR_B^*$, under the assumptions above, is to reduce X_B.

It is also interesting to note that MC_B will be unaffected as well: MC_B can be broken down into raw material costs, and processing costs. An increase in P_A^* will raise P_A, thereby raising the marginal raw material cost. However, the reduction in Q_B will reduce marginal processing costs. It can be shown that these two effects balance out, leaving total marginal cost unaffected.

The effects of an increase in the world price P_B^* of B will be analogous to those above: for the optimality conditions (4) and (5) to hold, X_A has to be reduced, H_A remains the same, H_B is reduced and X_B is increased. The derivations are similar to those above, and can be found in (Prastacos and Xafa, 1978). We only want to point out that a unit increase in P_B^* will leave unaffected the marginal revenues MR_A and MR_A^* of A, while it will increase the marginal revenues MR_B and MR_B^* and the marginal cost MC_B of B by the size of the exchange rate R.

We now turn our attention to the effects of changes in the exchange rate R. This is of special interest since, contrary to world prices, exchange rates can be affected by government policies. The major result here is that an increase in R (i.e., a depreciation of the domestic currency) will favor the production of the processed commodity over the primary product. This result confirms a similar empirical result in Bautista's study. It also implies that the policy of keeping the domestic currency overvalued, often followed by less developed countries with chronic balance of payments problems, is contradictory to the objective of promoting exports of processed goods as opposed to primary goods.

A change in R is conceptually equivalent to a change in both P_A^* and P_B^*: an increase in R will increase the foreign marginal revenues MR_A^* and MR_B^*, and, therefore, to retain optimality, will reduce the home consumptions H_A and H_B. The reduction in domestic sales will in turn raise the domestic prices of both products. However, even though X_B will certainly increase (as the result of an increase in Q_B and a decrease in H_B), the same is not necessarily true for X_A: H_A is reduced, but H_A+X_A is also reduced (since Q_B is increased), and, therefore, the net effect on X_A is unclear. We also note that a unit change in R will upset the values of all marginal revenues and costs: MR_A and MR_A^* will change by P_A^*, and MR_B MR_B^*, MC_B will change by P_B^*. The proof is similar to before and therefore omitted.

Finally, it is also useful to examine the total variations of each of the decision variables in terms of proportionate variations of the exogenous variables. We define $\hat{D}_i = \partial D_i/D_i$ as the proportionate change of decision variable D_i. In general, we have $\hat{D}_i = \Sigma_j \eta_{i,j} \hat{V}_j$, where $\eta_{i,j} = (\partial D_i/\partial V_j)/(D_i/V_j)$.

Using the results of Table 1, it can be shown that:

$$\hat{P}_A = [\hat{P}_A^* + \hat{R}] > 0 \quad , \quad \hat{P}_B = [\hat{P}_B^* + \hat{R}] > 0 \tag{8}$$

$$\hat{Q}_B = \frac{R}{2C_2Q_B} [P_B^*(\hat{P}_B^* + \hat{R}) - uP_A^*(\hat{P}_A^* + \hat{R})] \tag{9}$$

$$\hat{H}_A = \alpha_1[\hat{P}_A^* + \hat{R}] = \alpha_1\hat{P}_A < 0 \quad , \quad \hat{H}_B = \beta_1[\hat{P}_B^* + \hat{R}] = \beta_1\hat{P}_B < 0 \tag{10}$$

$$\hat{X}_A = -\left(\alpha_1 \frac{H_A}{X_A} + \frac{u^2 RP_A^*}{2C_2 X_A}\right)(\hat{P}_A^* + \hat{R}) - \frac{uRP_B^*}{2C_2 X_A}(\hat{P}_B^* + \hat{R}) \qquad (11)$$

$$\hat{X}_B = \left(\beta_1 \frac{H_B}{X_B} + \frac{RP_B^*}{2C_2 X_B}\right)(\hat{P}_B^* + \hat{R}) - \frac{uRP_A^*}{2C_2 X_B}(\hat{P}_A^* + \hat{R}) \qquad (12)$$

It is interesting to note that from (8) it follows that the percentage change of the domestic price is equal to the sum of the percentage changes in the exchange rate and the world prices. This result confirms a similar empirical observation made in (Bautista, 1978) for the coconut industry of the Philippines. Also, (10) follows directly from the definition of α_1 and β_1 as elasticities of domestic demand.

4. Extensions

We now relax the small country assumption. Assume that

$$X_A = f^*(R \cdot P_A^*, P_A, P_{CA}, E) \;,\; X_B = g^*(R \cdot P_B^*, P_B, P_{CB}, E)$$

where f^* and g^* are the export supply functions for A and B, respectively. We also assume that the domestic supply functions are given again by (1a) and (1b).

By following the methodology developed in section 3, it can be shown that similar optimality conditions hold. Solving for the log-linear case (i.e., assuming log-linear forms for f, g, f^*, g^*), we get

$$P_A = RP_A^* v_A^* / v_A \;,\; P_B = RP_B^* v_B^* / v_B \qquad (13)$$

$$Q_B = [R(P_B^* v_B^* - uP_A^* v_A^*) - C_1]/2C_2 \qquad (14)$$

where

$$v_x = 1 + (\alpha_x P_x / H_x)^{-1} \;,\; v_x^* = 1 + (\alpha_x^* P_x^* / X_x)^{-1} \qquad (15)$$

We note that the solution is structurally similar to that of the small country model. The same conclusion can be drawn for the comparative statics analysis. Detailed results appear in (Prastacos and Xafa, 1978).

References

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